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Study versus television

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Abstract

The great majority of studies on the effect of school quality on academic outcomes do not take account of changes in student choices concerning effort if school quality, e.g. class size, changes. We show that empirical estimates of the 'total' effect of changes in school quality could be quite different from the 'partial' effect holding other inputs (including student effort) constant. The main parameters governing this difference are the extent to which inputs in the education production function are substitutes or complements and how kinked is the benefit from a higher mark. The difference depends also on student ability, the student's distaste for effort and the curvature of the education production with respect to effort.

JEL classification: I21; I28

Keywords: Educational economics; Human capital; Effect of school quality; Student effort; Structural modelling; Education production function

1 Introduction

The majority of studies on the effect of 'school quality' on academic outcomes do not take account of changes in student choices concerning effort when school quality, e.g. class size or teacher quality, changes.[1] In particular, students might respond to changes in their school quality by adjusting the time and effort devoted to study (with a consequent change in leisure or market work). Thus, the 'partial' effects of school quality on academic outcomes corresponding to production function parameters may differ from empirical estimates of the 'total' effects (which include effects via effort response). This is similar to the distinction between production function parameters and average policy effects in Todd and Wolpin (2003) where family inputs are allowed to respond to changes in school quality.

From a policy point of view, both the total and partial effects of an increase in school quality are of interest. The total effect on student achievement is of course important for policy makers: An intervention aimed at improving student academic outcomes will typically be considered a success if the goal is achieved, and a failure otherwise (irrespective of specific mechanisms, including possible effort response). However, the partial effect on achievement (holding effort constant) is also important, since it provides knowledge of education production function parameters and the mechanisms through which an intervention works (or why it does not work). Furthermore, if students respond to an intervention which increases school quality by reducing time and effort devoted to study, benefit-cost analysis that only have total effects as benefit will underestimate 'full'

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benefits. These include, *inter alia*, more leisure, increased earnings from market work and less need for intervention by parents. In the formal model of this paper we focus on educational outcomes and student leisure, but benefits are much wider than that.

Empirical estimates of class size effects are often rather small and insignificant. Reasons for this may be non-random sorting of students into schools and that schools reduce class size when students are more 'disruptive' (Lazear 2001). Another explanation may be that students typically reduce effort in response to a reduction in class size.

A few papers model and estimate the response of parental inputs to changes in school quality. Houtenville and Conway (2008) consider a theoretical model in which student achievement depends on parental effort and school resources, and parents maximize utility, which is a function of student achievement, leisure and consumption, subject to time and budget constraints. In this model, an increase in school resources may induce parents to increase or reduce their effort depending on the form of the utility and production functions. Their empirical analysis indicates that parental effort and per-student spending have positive effects on student achievement, and that some measures of parental effort are affected negatively by per-student spending. However, the estimated effect of per-student spending on achievement is not affected by whether or not parental effort is included in the model. A similar result is found in Datar and Mason (2008): controlling for parental involvement does not change estimated effects of class size on test scores for children in kindergarten and first grade. Bonesrønning (2004) finds zero or positive effects on parental effort of reducing class size. Das *et al.* (2011) consider a dynamic household optimization model where child test scores depend on school and household inputs. Assuming that households make decisions regarding their own inputs before they know the amount of school inputs, they are only able to respond to anticipated changes in school inputs. Using data from Zambia and India the authors find that household school expenditure is reduced when anticipated school grants are increased, and that anticipated grants have no effect on student test scores whereas unanticipated grants have significant positive effects.

Only very few papers consider models of student response to changes in school quality. In the theoretical model of De Fraja *et al.* (2010) student effort, parental effort and school effort are simultaneously determined as a Nash equilibrium. In this very general model, a change in an exogenous variable, e.g. an increase in school resources, may increase or reduce the equilibrium level of effort of students (and parents and schools) depending on the form of the utility and education production functions and the values of exogenous variables. In their empirical analysis, the measure of student effort is based on (a factor analysis of) general attitude variables such as whether students like school, whether they think homework is boring and whether they want to leave school. The authors do not find any significant effect of class size on their measure of student effort, but they do find that student effort is reduced when 'school effort' is increased, where school effort is based on (a factor analysis of mainly) whether streaming and disciplinary methods are used. They find that student and parental effort are positively correlated (where parental effort is based on a factor analysis of mainly the teacher's opinion of parents' interest in their child's education) and that class size has no effect on parental effort. The attitudinal variables used by De Fraja *et al.* (2010) may be poor proxies for effort or time spent on homework, and they may not be expected to be much affected by (marginal) changes in school resources. Furthermore, the authors ignore the important issue of the endogeneity of class size and assume observed class size variation to be exogenous. These problems may explain why they do not find any effect of class size on their measure of student effort.

Using quasi-experimental variation in class size, a recent study estimates student and parental response to class size in grades 4-6 (Fredriksson P, Öckert N, Oosterbeek H: Inside the black box of class size effects: Behavioral responses to class size variation, unpublished). Their measures of student effort are time spent on homework and reading outside school. In their theoretical model (based on Albornoz F, Berlinski S, Cabrales A: Motivation, resources and the organization of the school system, unpublished), the education production function determines student skills as the product of student ability and student effort, so class size only affects student skills through student effort. They assume that student utility depends on student effort and 'rewards to effort' which are determined by parents' and teachers' utility maximizing behaviour. In their model, class size always has a negative effect on student effort, but a non-negative effect on parental effort. In this paper we consider a theoretical parametric model which includes a more general education production function and a more flexible student utility function, which depends on an educational outcome and effort, in order to analyse how student response to a change in school quality may depend on the values of important parameters of the production and utility functions. We ignore parental response to a change in school quality. This simplification is more appropriate when considering older students (e.g. 15-year-olds).

Some papers estimating the effects of course-specific (or subject-specific) school quality inputs on student academic outcomes provide indirectly an indication that students' change of effort in response to changes in school quality may be important. Aaronson *et al.* (2007) estimate the effect of teacher quality and find, e.g., statistically significant effects of mathematics teachers on mathematics test scores, but also significant effects of English teachers on mathematics test scores (and of mathematics teachers on English test scores). Heinesen (2010) estimates significant negative effects of subject-specific class size on examination marks in the same subject, but the results also indicate negative effects on marks in other subjects. One interpretation of the effects of subject-specific school inputs on student outcomes in other subjects is that they are due to spill-over effects between subjects induced by student reallocation of effort between subjects.

In this paper we consider simple parametric models with endogenous student effort and show that students' responses to changes in their school quality could imply quite large differences between total and partial effects on academic outcomes of changes in school quality and could lead to empirical estimates of total effects which are either larger or smaller than corresponding partial effects. The main parameters governing the sign of the difference between total and partial effects are shown to be the extent of substitution in the education production function and how kinked is the benefit from a higher mark. The absolute value of the difference also depends on the student's distaste for effort, the curvature of the 'production' of the final mark with respect to effort and the student's ability in the course of study. Our main conclusion is that reliable estimates of the partial effect of school quality on academic outcomes requires information on academic effort, including time use. This suggests a new round of data collection.

2 The basic idea

We begin with an illustration of the basic idea. Assume that an educational outcome (e.g. test scores or examination marks) is a concave function of student effort for a fixed level of school quality, and that student utility is increasing in the educational outcome and decreasing in effort. For a given level of school quality, students choose effort, and thereby (ignoring uncertainty) the educational outcome, to maximize utility. If school quality increases, education production possibilities increase for each level of effort. This is illustrated in Figure 1 for four different cases of production and utility functions. In each panel of Figure 1 the concave curves represent the education production functions before and after the increase in school quality (solid and dashed lines, respectively), and the convex curves are student indifference curves. The points marked by A are the initial utility maximizing choices of effort and outcome, and points marked by B are the corresponding choices after the increase in school quality. The 'partial' effect of an increase in school quality is given by the vertical shift of the production function at point A (i.e., holding effort fixed at its initial optimal level). However, the upward shift in the production function implies that students may obtain a higher outcome with less effort. This 'income effect' tends to reduce effort. The substitution effect may enhance the negative effort response if the marginal product of effort decreases when school quality increases, or it may work in the other direction if effort and school quality are complements in production. Of course, the optimal response of students also depends on the form of the utility function. The two left panels of Figure 1 illustrate cases where effort is reduced (more so in the upper panel) implying that the total effect on the educational outcome is smaller than the partial effect. In the upper right panel, effort is unchanged (no difference between total and partial effect), and in the lower right panel effort is increased (the total effect exceeds the partial effect).

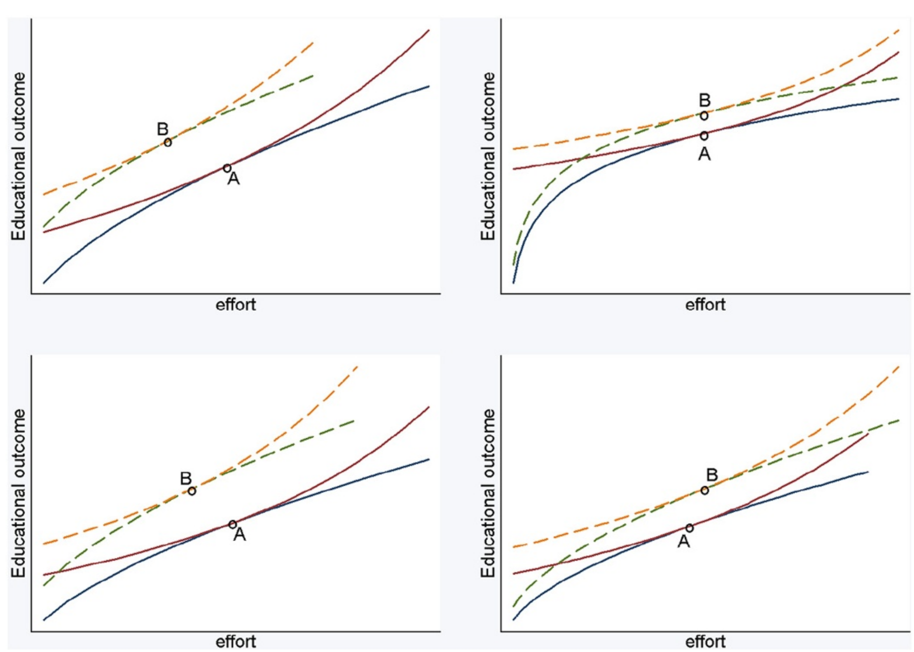

Figure 1 Education production relations between outcome and student effort for two different levels of school quality, and student indifference curves - four different cases of production and utility functions.

3 A parametric model

To fix ideas we consider a parametric model. We begin with the simplest case in which a student takes only one course.[2] The outcome is a mark y. This mark is the result of school quality, s, student ability, μ, and student effort, h. [3] To make our main points as cleanly as possible we ignore uncertainty and take a simple parameterisation for the production function:

$$y = s^{\lambda} + \mu h^{\eta} + \varrho s^{\lambda} \mu h^{\eta} \tag{1}$$

Student effort and school quality are normalized so that $0 < h < 1$ and $0 < s \leq 1$. The parameters η and λ capture the curvature in the output with respect to student effort and school quality, respectively, and ϱ measures the degree of substitution or complementarity of the two inputs. To ensure that production is increasing and concave in both we assume that $0 < \lambda < 1, 0 < \eta < 1, \varrho > -1$ and $\varrho \geq -(\bar{\mu})^{-1}$, where $\bar{\mu}$ is the maximum value of μ. If $\varrho < 0$ then s and h are substitutes in production, and if $\varrho > 0$ they are complements. To motivate this function, note that education production may be considered to consist of two learning processes, learning at school (represented by the term s^{λ}) and learning at home doing homework (represented by μh^{η}), and an interaction effect between the two processes represented by the last term in (1). The marginal effect of school resources may be smaller for well-prepared/high-ability students ($\varrho < 0$) if the primary goal of teaching is to ensure that all students obtain a basic level of skills. Also, the marginal effect of effort (and ability) may be smaller when school resources are high: when the learning process at school is more effective there may be smaller returns to effort at home to learning the curriculum. In conventional production functions including the Cobb-Douglas and CES functions, inputs are complements, i.e. the marginal product of one input (e.g. h) increases when the amount of the other input (s) is increased. However, as argued above, h and s may be substitutes in production, so that the marginal product of h is reduced when s increases, and *vice versa*.[4]

In empirical studies a parameter of interest is *the partial elasticity* of the outcome with respect to school quality, holding effort constant:

$$\epsilon = \frac{\partial \ln y}{\partial \ln s} \mid_h \tag{2}$$

Note that in the parameterisation (1) the partial elasticity is not independent of effort and student ability. The same is true for the corresponding derivative $\partial y / \partial s$, unless $\varrho = 0$. If effort is fixed then we can recover the partial elasticity from observing variation in y due to experimental variation in s.

Typically neither experimental nor non-experimental studies of effects of school quality have access to data on student effort or time use, and therefore the interpretation of estimated effects as education production function parameters presumes that effort is fixed. However, it is perfectly reasonable to assume that some students might respond to the change in constraints with adjustment in effort expended. To capture this, let preferences be represented by the utility function:

$$u = \frac{y^{1-\sigma}}{1-\sigma} + \delta \ln (1-h) \tag{3}$$

where the maximum time available for study is normalised to unity (as noted above).[5] The parameter δ captures the taste for leisure and the parameter σ (> 0) captures the curvature in the concern about the outcome; higher values of σ give a more kinked benefit function. That is, for higher values of σ the student will have a high return below a threshold value of y ('passing') and a low return above.

Denoting the optimal choices by $\left(\hat{h}, \hat{y}\right)$ *the total elasticity* is given by:

$$\hat{\epsilon} = \frac{d \ln \hat{y}}{d \ln s} \tag{4}$$

Just as the partial elasticity is a parameter of interest in empirical studies, so is the total elasticity, which may be recovered by observing variation in the outcome (including variation via effort response) due to experimental variation in s. However, in general, the total elasticity will not be equal to the partial elasticity ϵ. We define the *elasticity difference* as

$$\Delta = \hat{\epsilon} - \epsilon \tag{5}$$

The sign of the elasticity difference will depend on the sign of the *effort elasticity*, $\partial \ln \hat{h} / \partial \ln s$. Clearly the elasticity difference Δ will be positive ($\hat{\epsilon} > \epsilon$) if the student responds to the increase in school quality by putting in more effort, and it will be negative if the effort elasticity is negative. If s and h are substitutes in production ($\varrho \leq 0$) the effort elasticity and the elasticity difference are negative. But if they are complements ($\varrho > 0$) the marginal product of effort is increased when school quality increases, and therefore it may be optimal for the student to increase effort. Whether it is in fact optimal to increase effort depends on the size of ϱ and the other parameters of the model, especially the curvature of the benefit function with respect to the outcome (σ). Thus, even if s and h are complements it may still be optimal to reduce effort because of the 'income effect': an increase in s enables students to obtain a larger y with less effort (more leisure).

First, we show that the effort elasticity (and therefore the elasticity difference) is negative if $\sigma > 1$ or $\varrho \leq 0$. The marginal effect of school quality on effort is found by inserting (1) into (3) and differentiating the first-order condition:

$$\begin{aligned} \frac{\partial \hat{h}}{\partial s} &= -\frac{\partial^2 u / \partial \hat{h} \partial s}{\partial^2 u / \partial \hat{h}^2} \\ \frac{\partial^2 u}{\partial \hat{h} \partial s} &= \lambda s^{\lambda-1} \mu \eta h^{\eta-1} y^{-\sigma} \left[\varrho(1-\sigma) - \sigma y^{-1} \right] \\ \frac{\partial^2 u}{\partial \hat{h}^2} &= \mu \eta h^{\eta-2} (1 + \varrho s^{\lambda}) y^{-\sigma} \left[(\eta - 1) - \sigma y^{-1} \mu \eta h^{\eta} (1 + \varrho s^{\lambda}) \right] - \frac{\delta}{(1-h)^2} < 0 \end{aligned} \tag{6}$$

Thus, the sign of $\partial \hat{h} / \partial s$ is equal to the sign of $\partial^2 u / \partial \hat{h} \partial s$. It is obvious that $\partial \hat{h} / \partial s < 0$ if ($\varrho \leq 0$ and $\sigma < 1$) or if ($\varrho \geq 0$ and $\sigma > 1$). However, $\partial \hat{h} / \partial s$ is also negative when $\varrho < 0$ and $\sigma > 1$. Thus, when $\varrho < 0$ and $\sigma > 1$ we have

$$\frac{\partial^2 u}{\partial \hat{h} \partial s} < 0 \Leftrightarrow \varrho(1-\sigma) - \sigma (s^{\lambda} + \mu h^{\eta} + \varrho s^{\lambda} \mu h^{\eta})^{-1} < 0 \tag{7}$$

$$\Leftrightarrow s^{\lambda} + \mu h^{\eta} + \varrho s^{\lambda} \mu h^{\eta} < \frac{\sigma}{1-\sigma} \frac{1}{\varrho} \tag{8}$$

This inequality always holds given the assumed restrictions on the parameters. The RHS of the inequality tends towards its lower limit $\max(1, \overline{\mu})$ from above when $\sigma \to \infty$ and $\varrho = \max(-1, -1/\overline{\mu})$. It is easy to show that: (a) for $\varrho = \max(-1, -1/\overline{\mu})$ the LHS of (8) is always less than or equal to $\max(1, \overline{\mu})$; (b) when $\max(-1, -1/\overline{\mu}) < \varrho < 0$ the inequality

(8) also holds (the derivative of the LHS with respect to ϱ is smaller than the derivative of the RHS). Thus, $\partial\hat{h}/\partial s < 0$ if $\varrho \leq 0$ or $\sigma > 1$.

We now examine further the determinants of the direction and size of the elasticity difference Δ. Although simple, the parametric model does not yield closed form expressions for the elasticities of interest. We therefore have to resort to simulations to illustrate how they vary with the parameters. Without loss of generality we can take $\lambda = 0.4$.[6]

We take a grid over the values given in Table 1 (and calculate the elasticities at $s = 1$).[7] These values are, of course, wholly arbitrary and serve only to illustrate the variation in the elasticity difference. Since the maximum value of μ is here equal to 10, concavity is ensured when $\varrho \geq -0.1$. For each set of parameter values, the partial elasticity is calculated holding h fixed at the optimal level given the parameters and the initial level of school quality, whereas the total elasticity is calculated letting h adjust to its new optimal level induced by the increase in s.

With this range of parameter values the minimum and maximum of the elasticity difference Δ are -0.12 and 0.00, respectively. Thus, even when ϱ attains its maximum value of the grid (0.075) Δ is not positive for any combination of the other parameters within the grid of Table 1. Table 2 shows that the parameter values that induce the extreme values of Δ are very different. As expected, the largest negative value of Δ is obtained when s and h are strong substitutes in production (ϱ attains its minimum). Also, Δ is more negative if the benefit function is more curved (high σ), if the student has a higher taste for leisure (high δ), if the elasticity of output with respect to effort is high (high η), and/or if student ability (μ) is relatively low (although in this case not at its minimum).

When s and h are strong complements in production Δ may be substantially positive. This is illustrated in Table 3 where ϱ is fixed at 2. Here the largest negative value of Δ is obtained for about the same values of $(\mu, \delta, \sigma, \eta)$ as in Table 2 (except that μ is 2 instead of 3), whereas the largest positive value of Δ (0.055) is obtained for the same parameter values except that σ (the curvature of the benefit function) is at its minimum instead of its maximum.[8]

Within any school, we would expect that the parameters $(\mu, \delta, \sigma, \eta, \varrho)$ are heterogeneous. For example, how important it is to attain more than a simple 'passing' grade will vary from student to student, implying heterogeneity in the parameter σ. Moreover, the distributions of these parameters may not be independent. For example, high ability students (high μ) who aspire to further education may have a lower concern for simply passing.

4 Differential effects

The results of Summers and Wolfe (1977), Krueger (1999), Angrist and Lavy (1999), Browning and Heinesen (2007) and Heinesen (2010) indicate that reducing class size

Table 1 Simulation parameter values

Parameter	Minimum	Maximum	Grid step
μ	1	10	1
δ	0.5	2.0	0.1
σ	0.45	1.95	0.1
η	0.1	0.6	0.1
ϱ	−0.075	0.075	0.025

Table 2 Extreme values of the difference between total and partial elasticities

Partial	Total	Diff.	Parameters				
elasticity	elasticity	Δ	μ	δ	σ	η	ϱ
0.249	0.125	−0.124	3	2.0	1.95	0.6	−0.075
0.062	0.062	−0.000	10	0.5	0.45	0.1	0.075

has larger positive effects for students from disadvantaged backgrounds, and Heinesen (2010) also finds that low-ability students benefit significantly more than high-ability students. Aaronson *et al.* (2007) find that teacher-quality effects are relatively larger for lower-ability students.

The simple model described above is consistent with these findings since the total elasticity, $\hat{\epsilon}$, is decreasing in student ability, μ. The absolute value of the total effect of school resources on marks ($d\hat{y}/ds$) is also decreasing in μ for many combinations of values for the other parameters. When $\varrho = 0$ (i.e., s and h are neither substitutes nor complements in production), the derivative $\partial y/\partial s$ in the production function (1) holding h fixed does not depend on μ. However, the total effect $\partial\hat{y}/\partial s$ depends on μ because students respond to changes in school quality by changing effort. This is illustrated in Figure 2 which shows how $\hat{\epsilon}$ and $d\hat{y}/ds$ vary with μ in the model consisting of (3) and (1) where $(\delta, \sigma, \eta) = (1.0, 1.05, 0.4)$. In the lower part of the figure $\varrho = 0$, and the lower right panel shows that $d\hat{y}/ds$ is decreasing in μ. The lower left panel shows that the total elasticity decreases much more in μ. This is not surprising since y determined by the production function (1) holding h fixed and also the optimal value $\hat{y}$ are strongly increasing in μ. In the upper part of the figure $\varrho = -0.05$ (i.e., s and h are substitutes in production). Whereas the total elasticity varies with μ in much the same way in the two parts of the figure, $d\hat{y}/ds$ decreases much more in the upper part of the figure, where s and h are substitutes, than in the lower part.

Figure 3 illustrates differential effects with respect to the parameters of the utility function, i.e. the curvature with respect to the educational outcome (σ) and the taste for leisure (δ), given constant values of student ability ($\mu = 5$) and the curvature of the production function with respect to effort ($\eta = 0.4$). In the special case where $\varrho = 0$, school quality (s) and effort (h) are neither substitutes nor complements in production, and the derivative with respect to s in the production function, holding h constant, is equal to $\lambda s^{\lambda-1}$, i.e. independent of the initial levels of h and y. Assuming $\lambda = 0.4$ and $s = 1$ as above and considering a 10% increase of s, the partial effect on y is 0.04. This is represented by the horizontal line in the upper left panel of Figure 3, whereas the two downward sloping curves in this panel show the total effects for $\delta = 0.5$ (solid line) and $\delta = 2$ (dashed line), respectively. Whereas the partial effect is constant, the total effect is decreasing in

Table 3 Extreme values of the difference between total and partial elasticities when school quality and effort are strong complements (rho=2)

Partial	Total	Diff.	Parameters				
elasticity	elasticity	Δ	μ	δ	σ	η	ϱ
0.326	0.217	−0.109	2	2.0	1.95	0.6	2
0.299	0.354	0.055	2	2.0	0.45	0.6	2

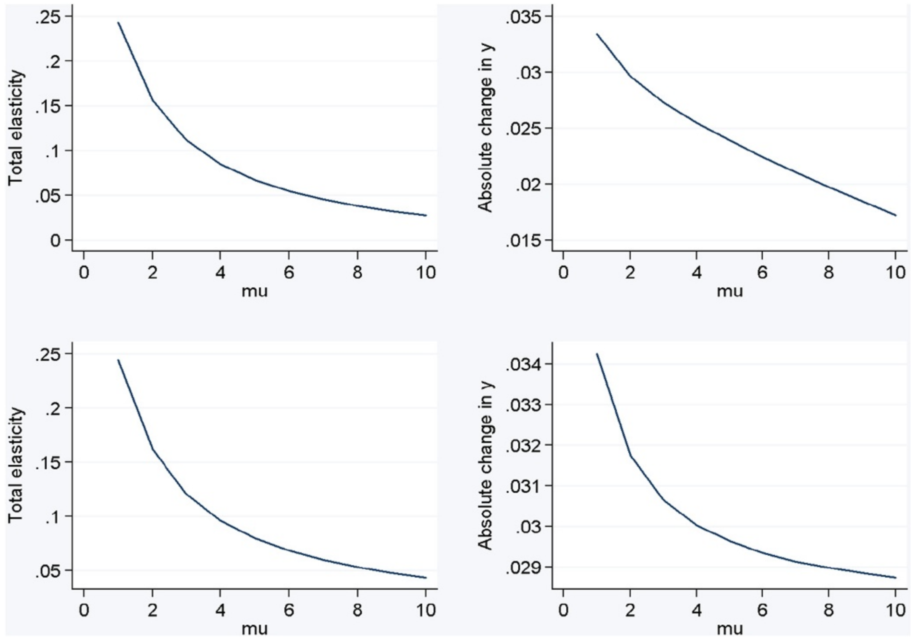

Figure 2 Total elasticity and absolute change in outcome when school resources are increased by 10%. The two upper panels are for rho = −0.05, the two lower panels are for rho = 0.

both σ and δ. When both σ and δ are small (about 0.5) the total effect is about 6% smaller than the partial effect; the difference is about 12% if instead δ is large (2); and it is about 40% if σ is large (1.95). The upper right panel of Figure 3 shows the corresponding relationship between the elasticity difference Δ and σ for the two extreme values of δ (and again for $\varrho = 0$). The numerical value of the elasticity difference increases in both σ and δ, and it is about -0.07 when both parameters are large (about 2) and in this case the partial elasticity is 0.16. The relation between the elasticity difference and the utility function parameters is almost the same when $\varrho = -0.05$, i.e. when s and h are substitutes in production; see the lower left panel. The lower right panel shows the relation when s and h are strong complements ($\varrho = 2$): when σ is small the elasticity difference is positive (h increases in response to the increase in s), and more so when δ is large; but when σ is large the elasticity difference is negative and large numerically (of about the same size as when $\varrho = 0$ or $\varrho = -0.05$).

5 A pass/fail mark

If in this simple model the curvature parameter in the utility function is above a threshold ($\sigma > 1$), students respond to an increase in school quality by *decreasing* their effort. Conversely, if $\sigma < 1$ (and $\varrho > 0$), students may in some cases respond by increasing their effort. To investigate in more detail the effect of the curvature of the benefit function we consider an extreme case in which:

$$u = \mathbf{I}\left(y \geq y^*\right) + \delta \ln\left(1 - h\right) \tag{9}$$

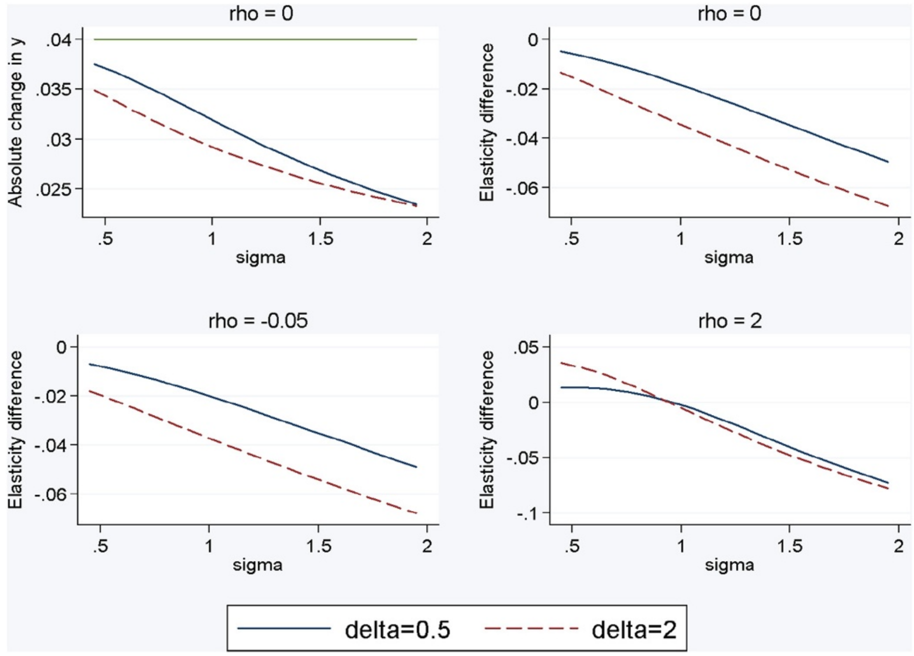

Figure 3 Differential effects with respect to parameters of the utility function: Total and partial effects and elasticity differences.

where $\mathbf{I}\,(y \geq y^*)$ is an indicator function that takes value unity if $y \geq y^*$ and zero otherwise (with $y^* > s^\lambda$). This corresponds to a pass/fail mark. In this case students will either set:

$$h = 0 \Rightarrow u = 0$$
$$h = \left(\frac{y^* - s^\lambda}{\mu(1 + \varrho s^\lambda)}\right)^{\frac{1}{\eta}} \Rightarrow u = 1 + \delta \ln\left(1 - \left(\frac{y^* - s^\lambda}{\mu(1 + \varrho s^\lambda)}\right)^{\frac{1}{\eta}}\right) \tag{10}$$

where we assume that the passing grade is attainable for some feasible level of effort: $y^* < s^\lambda + \mu(1 + \varrho s^\lambda)$. The student chooses to exert effort if:

$$\delta \leq -\left(\ln\left(1 - \left(\frac{y^* - s^\lambda}{\mu(1 + \varrho s^\lambda)}\right)^{\frac{1}{\eta}}\right)\right)^{-1} \tag{11}$$

This illustrates that, *ceteris paribus*, a pass grade is more likely if complementarity in production, student ability or school quality are high or if the student has a low taste for leisure.[9] For a given level of school quality we have three groups of students: bad fails; marginal fails (students who failed but were close to choosing to pass) and passes. If we increase school quality then the bad fails continue to exert no effort and fail, the marginal fails increase their effort (the level given in (10) with the new level of s) and pass students reduce their effort and still pass. Thus we have three different responses to the policy change: negative, zero and positive.

6 More than one course of study

As we discussed in the introduction, results in papers using course- or subject-specific variation in school quality indicate that students' effort response may be important. Thus, Aaronson *et al.* (2007) find significant effects of, e.g., both math and English teachers on math test scores, and Heinesen (2010) finds significant negative effects of course-specific class size on marks in the same course, but also indication of negative effects on marks in other courses. One possible mechanism which may explain these cross-course effects (or spill-over effects between courses) is student reallocation of effort between subjects. This may be illustrated by an extension of the above model framework to the more general case with more than one course of study. For simplicity, consider the case with two courses. We assume that the utility function is additive in the two outcomes (with the same curvature parameter) and leisure:

$$v = \frac{y_1^{1-\sigma}}{1-\sigma} + \frac{y_2^{1-\sigma}}{1-\sigma} + \delta \ln (1 - h_1 - h_2) \tag{12}$$

where y_i and h_i are the outcome (a mark) and student effort in subject i, respectively.

We allow student ability (μ_i) to differ between subjects, but for simplicity we assume that the other parameters in the two subject-specific production functions are identical, and we assume their form to be similar to (1):

$$y_i = s_i^{\lambda} + \mu_i h_i^{\eta} + \varrho s_i^{\lambda} \mu_i h_i^{\eta}, \quad i = 1,2 \tag{13}$$

Denoting the optimal choices by $(\hat{h}_1, \hat{h}_2, \hat{y}_1, \hat{y}_2)$ we may consider four total elasticities, namely elasticities of the two outcomes with respect to each of the two school quality inputs:

$$\hat{\epsilon}_{ii} = \frac{d \ln \hat{y}_i}{d \ln s_i}, \quad i = 1,2 \tag{14}$$

$$\hat{\epsilon}_{ij} = \frac{d \ln \hat{y}_i}{d \ln s_j}, \quad i,j = 1,2, \quad i \neq j \tag{15}$$

The two total 'own resource' elasticities in (14) consist of a direct effect of increased school resources in subject i on academic outcome in the same subject and an indirect effect through changed effort in subject i. The two total cross-elasticities (15) are different from zero if an increase in school resources in one subject induces students to change effort in the other subject. The partial own-resource elasticities, $\epsilon_{ii} = \partial \ln y_i / \partial \ln s_i$, consist of only the direct effect, holding effort fixed. The partial cross-elasticities, $\epsilon_{ij} = \partial \ln y_i / \partial \ln s_j, i \neq j$, are zero. The sign of each of the four elasticity differences $\Delta_{ij} = \hat{\epsilon}_{ij} - \epsilon_{ij}$ $(i,j = 1,2)$ is equal to the sign of the corresponding effort elasticity $\partial \ln \hat{h}_i / \partial \ln s_j$.

As in the model with only one course of study, the signs of the elasticity differences are mainly determined by the signs of σ and ϱ. Thus, we now show that $d\hat{h}_i/ds_i < 0$ and $d\hat{h}_i/ds_j > 0$ $(i,j = 1,2;\ i \neq j)$ if $\sigma > 1$ or $\varrho \leq 0$. To see this, we insert the production functions (13) into the utility function (12) and differentiate the first-order conditions:

$$\begin{aligned}
\frac{\partial \hat{h}_i}{\partial s_i} &= -\frac{\partial^2 v}{\partial \hat{h}_i \partial s_i} \frac{\partial^2 v}{\partial \hat{h}_j^2} / D \\
\frac{\partial \hat{h}_i}{\partial s_j} &= \frac{\partial^2 v}{\partial \hat{h}_j \partial s_j} \frac{\partial^2 v}{\partial \hat{h}_i \partial \hat{h}_j} / D \\
D &= \frac{\partial^2 v}{\partial \hat{h}_1^2} \frac{\partial^2 v}{\partial \hat{h}_2^2} - \left(\frac{\partial^2 v}{\partial \hat{h}_1 \partial \hat{h}_2} \right)^2
\end{aligned} \tag{16}$$

where

$$\frac{\partial^2 v}{\partial \hat{h}_i \partial s_i} = \lambda s_i^{\lambda-1} \mu_i \eta h_i^{\eta-1} y_i^{-\sigma} [\varrho(1-\sigma) - \sigma y_i^{-1}]$$
$$\frac{\partial^2 v}{\partial \hat{h}_i^2} = A_i - \frac{\delta}{(1-h_1-h_2)^2}, \quad \frac{\partial^2 v}{\partial \hat{h}_1 \partial \hat{h}_2} = -\frac{\delta}{(1-h_1-h_2)^2}$$
$$A_i = \mu_i \eta h_i^{\eta-2}(1+\varrho s_i^{\lambda}) y_i^{-\sigma} [(\eta-1) - \sigma y_i^{-1} \mu_i \eta h_i^{\eta}(1+\varrho s_i^{\lambda})] \tag{17}$$

Since $A_i < 0$, we have $\partial^2 v/\partial \hat{h}_i^2 < 0$. Furthermore, $D = A_1 A_2 - (A_1 + A_2)\delta/(1-h_1-h_2)^2 > 0$. Thus, $\partial \hat{h}_i/\partial s_i < 0$ and $\partial \hat{h}_i/\partial s_j > 0$ iff $\varrho(1-\sigma) - \sigma y_i^{-1} < 0$, and this inequality holds if $\sigma > 1$ or $\varrho \leq 0$ by arguments similar to the one-course case.

If we take a grid over the same values of the parameters as given in Table 1, where now both μ_1 and μ_2 vary between 1 and 10, we obtain the extreme values of the elasticity differences $\Delta_{11} = \hat{\epsilon}_{11} - \epsilon_{11}$ and $\Delta_{12} = \hat{\epsilon}_{12}$ and the associated parameters shown in Table 4.[10] The 'own resource' elasticity difference Δ_{11} has extreme values −0.12 and −0.00, and the same is true for Δ_{22} (not shown in the table since the model is symmetric in the two courses). The elasticity difference Δ_{11} (and Δ_{22}) vary with the parameters in basically the same way as Δ in the one-course case (except for the dependence on ability in the other subject): The extreme negative value is obtained when σ, δ and η are at their maximum, when ϱ is at its minimum, and when ability in the two subjects is low (although not at the minimum); the maximum value (zero) is obtained when when σ, δ and η are at their minimum, when ϱ is at its maximum, and when ability is at its maximum in the same subject and at its minimum in the other subject. The two cross elasticities $\hat{\epsilon}_{12}$ and $\hat{\epsilon}_{21}$ have extreme values −0.00 and 0.05. The extreme positive value of $\hat{\epsilon}_{12}$ is obtained when σ, η and μ_1 are at their maximum values, and when ϱ, δ and μ_2 are at their minimum values. Thus, the elasticity of outcome in one subject with respect to school resources in the other subject is high when ϱ is negative and numerically large, σ is high, ability in the first subject is high and ability in the other subject is low. The minimum of $\hat{\epsilon}_{12}$ is obtained when $\varrho = 0$ and when σ, δ, μ_1 and μ_2 are high, and η is low.

Table 5 illustrates that when school quality and student effort are strong complements in production ($\varrho = 2$ in the table) then the elasticity difference of Δ_{11} may be substantially positive (as for Δ in the model with a single subject) and $\hat{\epsilon}_{12}$ may be substantially negative. The maximum of Δ_{11} is obtained when σ and δ are small and η and μ_1 and μ_2 are high. The minimum of $\hat{\epsilon}_{12}$ is obtained for the same parameter values, except that μ_1

Table 4 Extreme values of differences between total and partial elasticities in model with two courses of study

Partial elasticity	Total elasticity	Diff.	Parameters					
			μ_1	μ_2	δ	σ	η	ϱ
ϵ_{11}	$\hat{\epsilon}_{11}$	Δ_{11}						
0.253	0.131	−0.121	2	2	2.0	1.95	0.6	−0.075
0.049	0.049	−0.000	10	1	0.5	0.45	0.1	0.075
ϵ_{12}	$\hat{\epsilon}_{12}$	Δ_{12}						
0.0	−0.001	−0.001	9	10	2.0	1.95	0.1	0.000
0.0	0.052	0.052	10	1	0.5	1.95	0.6	−0.075

Table 5 Extreme values of differences between total and partial elasticities in model with two courses of study when school quality and effort are strong complements (rho = 2)

Partial elasticity	Total elasticity	Diff.	Parameters					
			μ_1	μ_2	δ	σ	η	ϱ
ϵ_{11}	$\hat{\epsilon}_{11}$	Δ_{11}						
0.237	0.169	−0.068	10	10	0.5	1.95	0.6	2
0.210	0.308	0.098	10	10	0.5	0.45	0.6	2
ϵ_{12}	$\hat{\epsilon}_{12}$	Δ_{12}						
0.0	−0.069	−0.069	3	10	0.5	0.45	0.6	2
0.0	0.032	0.032	10	1	0.5	1.85	0.6	2

is small. However, Table 5 also shows that when σ is large, Δ_{11} may be substantially negative and $\hat{\epsilon}_{12}$ may be substantially positive even when s and h are strong complements in production.

Figure 4 shows how the total elasticities $\hat{\epsilon}_{ij}$ and the absolute change in the outcomes $D\hat{y}_{ij}$, which is the increase in $\hat{y}_i$ when s_j is increased by 10%, vary with ability in subject 1 when $(\mu_2, \delta, \sigma, \eta, \varrho) = (5, 0.5, 1.95, 0.6, -0.05)$. Thus, the values of δ, σ and η are chosen so that the cross elasticities are relatively large. The own-subject elasticity $\hat{\epsilon}_{11}$ and the absolute change $D\hat{y}_{11}$ are decreasing in μ_1 corresponding to the results in the one-subject case. The other own-subject elasticity $\hat{\epsilon}_{22}$ (and $D\hat{y}_{22}$) do not depend much on μ_1, while the cross elasticity $\hat{\epsilon}_{21}$ (and $D\hat{y}_{21}$) are decreasing in μ_1, and $\hat{\epsilon}_{12}$ (and $D\hat{y}_{12}$) are increasing in μ_1. The figure illustrates that the cross elasticities may be rather large compared to the own-subject elasticities. For instance, when $\mu_1 = 5$ the cross elasticities are about one third of the own-subject elasticities. Thus, when school resources in one course is changed, this may have substantial effects on outcomes also in other courses, and the mechanism

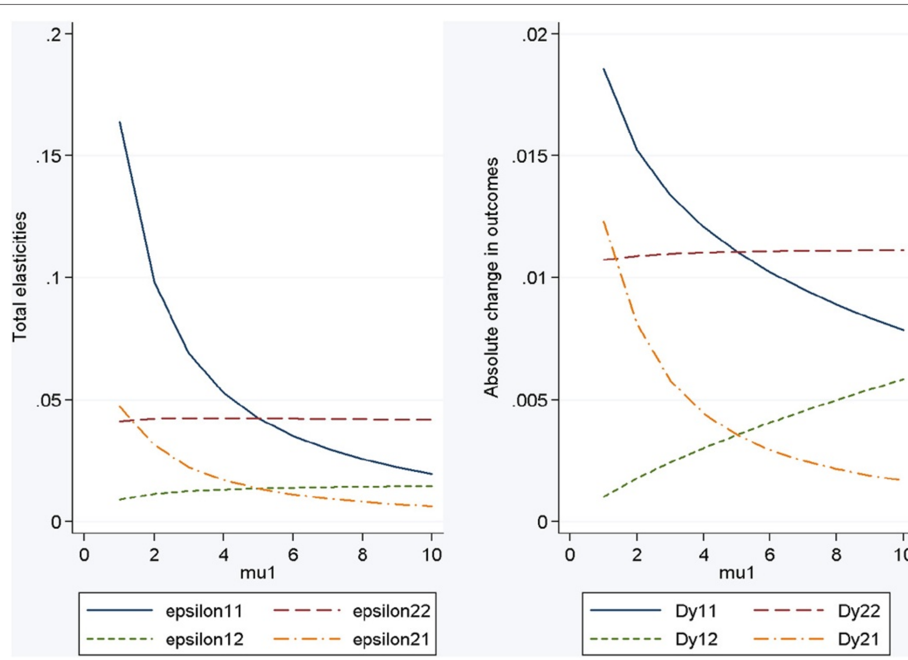

Figure 4 Total elasticities and absolute change in outcomes when school resources are increased by 10%.

behind these cross effects in this simple model is students' reallocation of effort between subjects: When school resources in one course increase it may be optimal for students to reduce effort in this course and increase leisure and effort in the other course. In Figure 4 $\varrho = -0.05$ so that s_i and h_i are assumed to be substitutes in production. Setting ϱ equal to zero produces a rather similar figure although the cross elasticities are a little smaller compared to the own-course elasticities (for $\mu_1 = 5$ the ratio is 0.27). Assuming inputs to be strong complements in production implies that the ratios of cross elasticities to own-course elasticities are smaller.

7 Conclusion

Typically, studies on the effect of school quality on academic outcomes do not take account of students' responses regarding academic effort or time use. Applying simple parametric models we have shown that students' effort or time-use responses to changes in school quality may cause large differences between the total elasticity of changes in school quality and the partial education production function elasticity (holding student effort constant). The main parameters determining the sign of the elasticity difference are the extent of substitution between effort and school quality in the production function and how kinked is the benefit from a higher mark in the student utility function. If effort and school quality are substitutes in production and/or if there is a marked kink in the benefit function, students will tend to reduce effort when school quality is increased implying that the total elasticity is smaller than the partial elasticity. The value of the elasticity difference also depends on the student's distaste for effort, the curvature of the production function with respect to effort and the student's ability in the course of study. In a model with two courses of study we have shown that an increase in school resources in one course may reduce effort in that course, implying that the total 'own-resource elasticity' is smaller than the partial own-resource elasticity, but increase effort in the other course implying a positive (total) 'cross elasticity'.

Our main conclusion is that reliable estimates of partial effects (based on education production function parameters holding student effort constant) of school quality on academic outcomes require - in addition to exogenous variation in school quality - information on academic effort and/or time use. This suggests a new round of data collection.

It is important to note that our models are very simple in several respects. For instance, we focus on student effort response to a change of school quality and ignore parental responses. Parental response is presumably very important for younger students. Thus, our model is mostly relevant for older students (e.g. 15-year-olds). Another limitation of our model is the assumption that school quality affects only the production function, but not the student utility function. An important aspect of school (and teacher) quality is to motivate and monitor students better, inducing them to put more effort in their school work. Thus, school quality may affect the parameters of the student utility function, but our model ignores this mechanism. In our model with two courses of study we focus on mandatory courses and spill-over effects between these. It would be interesting to extent the model to enable analysis of effects of changes in school quality and other interventions on the choice between optional courses, and academic outcomes given this choice (for instance the major choice and GPA at university, see e.g. Arcidiacono *et al.* 2012).

8 Endnotes

[1]Studies on the effect of class size include, inter alia, Hanushek (1996), Krueger (1999, 2003), Angrist and Lavy (1999), Case and Deaton (1999), Hoxby (2000), Krueger and Whitmore (2001), Heinesen (2010) and Fredriksson et al. (2013). Studies on the effect of teacher quality include Rockoff (2004), Rivkin et al. (2005), Aaronson et al. (2007) and Clotfelter et al. (2007a, 2007b, 2010). Other measures of school quality used in the literature include expenditure per student and the teacher-student ratio (see e.g. the surveys in Hanushek 1996, Card and Krueger 1996, and Betts 1996) and the number of teacher hours per student (Browning and Heinesen 2007).

[2]In the vastly simplified framework below we consider only a school that has one class. A more general model would distinguish between class quality and school quality and allow for student selection based on within school quality differences.

[3]We use Greek letters to denote preference and prodcution parameters and Latin letters to denote choice variables. Thus s is the choice of the school funding authorities. As discussed in the Introduction, we ignore parental effort as an input in the production function which means that the model is more relevant for older students.

[4]Houtenville and Conway (2008) note that school resources and parental inputs may be substitutes in education production.

[5]It is straightforward to allow for alternative uses of time such as market work. This complicates the notation and analysis without adding much of significance to the main points.

[6]Results are qualitatively the same for other values of λ.

[7]Choosing other values of s produces qualitatively similar results.

[8]The production functions and indifference curves of Figure 1 are based on the parametric model above and the four sets of parameters of Tables 2 and 3 (the upper panels of Figure 1 correspond to Table 2 and the lower panels to table 3).

[9]It is easily shown that $\frac{\partial}{\partial s}\left(\frac{y^* - s^\lambda}{\mu(1+\varrho s^\lambda)}\right) < 0 \Leftrightarrow 1 + \varrho y^* > 0$, and that this inequality holds because of the restriction $\varrho \geq -1/\overline{\mu}$.

[10]The elasticities are calculated at $\lambda = 0.4$ and $s_1 = s_2 = 0.5$.

Competing interests

The IZA Journal of Migration is committed to the IZA Guiding Principles of Research Integrity. The authors declares that they have observed these principles.

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